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Application of Kolmogorov-Smirnov Test to Evaluation of Population Distrbution by Age in Single Years

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This study examines the evaluation of population distribution by sex and age in single years. The aim is to assess the validity of the assumption that when data on population by age in single years is correctly reported then the population is uniformly distributed among end digits. The data used is a secondary data derived from the census populations reported by age in single years from Nigeria, Ghana, Indonesia and United State of America. Measures of digit preference are computed using the Ramachandran Index (RI), one of the most commonly algebraic methods of evaluation of age data in single years. The Kolmogorov-Smirnov test was used to test the Goodness-of-Fit of the discrete uniform distribution to the distribution of population among end digits in the selected countries. The results of the analyses show that the null hypotheses were all rejected at 1% level of significance, indicating that the distribution of population among the 10 end digits is not uniform for both sexes in all the selected countries. In other words, the distribution of population by age in single years in none of the countries studied is completely free from digit preference/avoidance. The study therefore, recommends that the age data should be adjusted for the error of digit/avoidance before using them for further estimation of demographic parameters or at least used with caution.

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Keywords: Population by age in single years; digit preference; data evaluation; uniform distribution; Kolmogorov Smirnov test.

1 Introduction

In all demographic enquiries, data are collected on many items in censuses and sample surveys.

Age and sex are two of the few items on the basis of which data are collected, tabulated, analyzed and adjusted in all demographic enquiries. Moreover, such phenomenon like fertility, mortality and migration are related to age and sex. Socio-economic characteristics, like nuptiality, education, occupation and employment are also highly related to age and sex. These data collected and tabulated, are not without errors. The type of errors in any demographic data depend on the source of the data among other factors listed by Ramachandran (1989). In Sub-Saharan African countries and most developing countries, demographic data are derived mainly from censuses and sample surveys. Errors most commonly found in age and sex data from these sources include age misreporting (digit preference, age shifting across critical age boundaries and exaggeration) which are associated with age in single years. Naturally, it is not possible to eliminate every error completely from any data. Teklu (1989) and Venkatacharya (1989) have noted that the quality of estimates of vital rates derived by indirect methods depends largely on the quality of data from which they were derived. Therefore, adequate knowledge of types and degree of errors in data helps an analyst to know the amount of confidence to place on estimates from them and serves as an aid to improved data quality in future surveys.

Data evaluation and error detection methods for population reported by sex and age in single years most commonly in use are the Whipple's Index, Digit Preference Index, Myers' Index and Ramachandran Index. Many authors have used them to evaluate the qualities of age – sex data from different countries. These include those of Kpedekpo, (1982) and Newell, (1988) gave reasons for the occurrence of these age distortions in developing countries to include high illiteracy rates, ignorance of age in the sense of the completed number of years, deliberate misstatement, and inability to understand the question asked. Mukherjee and Mukhopadhyay (1988) in their study using Turkish census data found that age heaping occurs in terminal digits "0" and "5". Kabir and Chowdhury (1981) in their analysis of census data of Bangladesh found errors in age reporting due to digit preference and there were strong tendency to report ages ending with "0" and "5", with subsidiary heaping at ages ending with "8" and "2" respectively. Gershon (2013) in his study of evaluation and adjustment of age sex data of the population and housing census of Ghana 2000 and 2010, revealed that age heaping occurred at digits 0 and 5. Nagi, Stockwell and Snavley (Nagi et al., 1973) revealed that age heaping is a major source of inaccuracy in the age statistics in many of the developing nations on the African continent, particularly among Islamic populations. The phenomenon was found to be more pronounced among women than men, and it tends to increase with age. Aimee and Samuel (1991) concluded that misreporting is most severe at an older age. They found evidence of very pervasive overstatement of age at advanced ages. The evidence of increasing age misstatement with old age is consistent with the observation that literacy rates have also declined with age, since age misstatement is associated with literacy and low educational attainment. Nwogu (2006; 2011) used graphical and algebraic method to show that the quality of age and sex data in the 1963 and 1991 Nigerian censuses, as well as the 1981/82 NFS, 1990, 1999 and 2003 NDHS for Nigeria, is quite low. Also that age preference for end digits 0 and 5, avoidance of the end digits 1, 3, 7, and 9 were pronounced in all surveys. So also was the problem of Age Shifting. Ohaegbulam (2015) also assessed the quality of age-sex data from 1991 and 2006 Nigerian population censuses, using some conventional techniques of evaluating demographic data quality. His results showed that there are obvious preferences for ages ending with end digits 0 and 5, while other end were avoided in the two censuses, this bias being more pronounced for females than males. (Nwogu et al., (2018) has equally demonstrated that the pattern of errors noticed in the total census population of Nigerian by age and sex also persisted across the census data by state after evaluation. Udoh, (2020); Abbani et al., (2021), (Onwuka et al., (2021), and (Okafor, 2018); Okoro, (2023) also arrived at similar forms of age misreporting.

In using these methods for the evaluation of population data by age in single years it was assumed that when data is correctly reported the population is uniformly distributed among the ten end-digits. However, in all these applications no attempt was made to assess the adequacy of this assumption. Therefore, the ultimate objective of this study is determine the goodness of fit of the discrete uniform distribution to the observed distributions of populations by age in single years among the ten end digits. Specifically, the study (i) evaluated the populations by age in single years (ii) applied the Kolmogorov-Smirnov test to assess the Goodness-of-Fit of the discrete uniform distribution to the distribution of populations among end digits in the selected countries.

2 Methodology

The data for this study is a secondary data derived from the censuses conducted in four selected countries. The four countries selected for this study include; Nigeria (1991 and 2006), Ghana (2000 and 2010), Indonesia (2000 and 2010) and USA (2000 and 2010). The data on the distribution of household population by sex and age in single years was retrieved from the United Nations database (http//data.un.org/data) (United Nations, n.d.).

The algebraic methods most commonly used for measuring digit preference/avoidance include; Whipple's Index (WI), Digit Preference Index (DPI), Myer's Index (MI) and Ramachandran Index (RI). Their computations are based on the assumption that when data on age in single years are correctly reported, the population is uniformly distributed among the ten end digits. However, Whipple's Index provides measures of digit preference/avoidance only for digits 0 and 5 in the age range 23 to 62. Digit Preference Index (DPI) provides measures of preference/avoidance for all end digits but it does not take into account the decreasing nature of population with the age. That is the population at younger ages receive greater weight than the older ages which makes the index biased. Myers' Index try to address the problem of DPI by introducing the blended population by introducing weighting of the end-digits. However, the differential weighting of the end-digits used in computing the blended population does not take into consideration differences in age structure of different populations. Thus, Myers' Index cannot be used to compare qualities of age data reported in single years for two or more countries with different population structures.

Ramachandran index (RI), adopted in this study as a measure of digit preference/avoidance, addresses the problems of the Whipple's index and the problem of differential weighting of the end-digits associated with the Myer's index. It provides measures of digit preference/avoidance for all end digits and takes into consideration the differences in age structure of different populations.

The Ramachandran index (RI) is defined as

$$RI = \sum_{i=0}^{9} \left| \% B_v(i) - 10 \right| \tag{2.1}$$

where, for the ith end-digit (i = 0, 1, 2, …, 9),

$\left| \% B_v(i) - 10 \right|$ is the absolute deviation of $\% B_v(i)$ from 10.

$$\% B_v(i) = \frac{B_v(i)}{\sum B_v(i)} \times 100 \tag{2.2}$$

is the percentage blended value,

$$B_v(i) = (i+1)\,\phi_{1i} + [10 - (i+1)]\,\phi_{2i} \tag{2.3}$$

is the blended value,

$$\phi_{1i} = \frac{P_{1i}}{\sum_{i=0}^{9} P_{1i}} \tag{2.4a}$$

is the proportion of the total population reported with the end-digit i within the range 10 – 59 years and

$$\phi_{2i} = \frac{P_{2i}}{\sum_{i=0}^{9} P_{2i}} \tag{2.4b}$$

is the proportion of the total population reported with the end-digit i within the range 20 – 69 years.

When data is correctly reported, each end-digit is expected to receive 10 percent of the total population. Therefore, the absolute difference $|\%_0 B_v(i) - 10|$ is expected to be zero for all end-digit. The Ramachandran index (RI) is, therefore an estimate of the minimum proportion of persons for whom age with an incorrect final digit is reported Shryock and Siegel (1976). However, the Ramachandran index (RI), like the other measures, does not provide a test of goodness-of-fit of the discrete uniform distribution to the observed population distribution among end-digits.

2.1 The Kolmogorov-Smirnov Test

In other to test the hypothesis that the population is uniformly distributed among the ten end-digits, the Kolmogorov-Smirnov has been proposed. Recall if $X_1, X_2 \ldots X_n$ is a random sample of size n from a population with distribution function $F_0(x)$ and $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ are the corresponding order statistics. The empirical distribution function $S_n(x)$ is defined as

$$S_n(\omega) = \begin{cases} 0 & X_{(1)} < \omega \\ \dfrac{k}{n} & X_{(k)} \leq \omega < X_{(k+1)} \\ 1 & X_{(n)} \leq \omega \end{cases} \tag{2.5}$$

And the Kolmogorov-Smirnov test statistic is given as

$$D_{cal} = Sup\,(Max)|F_0(x) - S_n(x)| \tag{2.6}$$

Where D_{cal} is the maximum absolute difference between the theoretical distribution function ($F_0(x)$) and the empirical distribution function ($S_n(x)$). Under the null hypothesis that the sample is from the population with distribution function $F_0(x)$, absolute difference between the theoretical distribution function ($F_0(x)$) and the empirical distribution function ($S_n(x)$) is expected to be zero. The more the absolute difference differs from zero the more the hypothesis is negated. Therefore the Kolmogorov-Smirnov test focuses on the maximum absolute difference between the theoretical distribution function ($F_0(x)$) and the empirical distribution function ($S_n(x)$) (D_{cal}). Therefore, the null hypothesis is rejected at α level of significance if $D_{cal} > D_n^{\alpha}$ or accepted otherwise.

This test was used to decide if a sample comes from a population with the specific distribution. In the present circumstance the theoretical distribution function is the cumulative discrete uniform distribution while the empirical distribution function is the cumulated populations at different end digits.

3 Results and Discussion

The deviations of the percentage blended values ($\%_0 B_v(i)$) from Ramachandran index from 10 are shown in Table 1 for all end-digits in the selected countries. As Table 1 shows, the digits that appear to be most preferred are 0 in all the countries and 5 in all the countries except the US. The digit most avoided is 9 except in Indonesia second census and the US. In the US, the pattern of preference and avoidance appear slightly different. The US population appears to show preference for end digits 0 to 3 and avoidance of others.

Table 1. Deviation of % $B_v(i)$ from 10

Digit (i)	Country															
	Nigeria				Ghana				Indonesia				USA			
	1991		2006		2000		2010		2000		2010		2000		2010	
	M	F	M	F	M	F	M	F	M	F	M	F	M	F	M	F
0	22.76	26.64	19.91	20.90	9.90	12.170	8.40	9.15	6.93	7.17	2.85	2.79	1.00	0.80	0.89	0.72
1	-4.63	-5.60	-4.44	-4.87	-2.97	-3.682	-1.25	-1.89	-0.57	-0.80	0.25	0.12	0.37	0.27	0.30	0.22
2	-1.64	-2.35	-0.96	-1.19	1.09	0.940	0.77	0.65	-0.42	-0.61	-0.05	-0.12	0.36	0.28	0.28	0.21
3	-4.42	-5.14	-3.87	-4.14	-1.60	-2.230	-1.33	-1.42	-1.60	-1.58	-0.67	-0.61	0.11	0.08	0.08	0.06
4	-5.19	-5.75	-4.29	-4.67	-2.05	-2.370	-1.51	-1.67	-1.34	-1.14	0.01	0.18	-0.31	-0.32	-0.28	-0.31
5	9.65	10.10	7.51	8.11	4.87	5.310	2.99	3.44	3.13	3.34	0.70	0.86	-0.06	-0.08	-0.08	-0.09
6	-4.55	-4.97	-3.76	-4.11	-1.73	-1.880	-1.66	-1.74	-1.88	-1.98	-0.89	-0.81	-0.40	-0.33	-0.33	-0.26
7	-4.17	-4.92	-3.87	-3.91	-3.02	-3.540	-2.01	-2.07	-1.04	-1.24	-0.26	-0.39	-0.30	-0.21	-0.23	-0.15
8	-1.96	-1.64	-1.44	-1.09	-0.67	-0.540	-0.65	-0.61	-1.39	-1.43	-1.19	-1.31	-0.43	-0.32	-0.33	-0.23
9	-5.86	-6.38	-4.79	-5.02	-3.59	-4.220	-3.77	-3.83	-1.88	-1.79	-0.76	-0.71	-0.40	-0.22	-0.30	-0.16

In order test the significance of the observed preference/avoidance the Kolmogorov-Smirnov test was applied to the distribution of the populations of the selected countries according to the ten end-digits. The empirical distributions ($S_n(x)$) derived from the Ramachandran Index (RI) and theoretical distributions ($F_0(x)$) based on the discrete uniform distribution are shown in Appendices A to D while the corresponding $D_{cal} = Sup\,(Max\,)\left|F_0(x) - S_n(x)\right|$ are shown in Table 2 for the selected countries.

Table 2. Computed $D_{cal} = Sup\,(Max\,)\left|F_0(x) - S_n(x)\right|$ by country, year and sex

Country/Year	Sex		D_{100}^{α}	
	Male	**Female**	$\alpha = 0.05$	$\alpha = 0.01$
Nigeria			0.136	0.163
1991	22.76	26.64		
2006	19.91	20.90		
Ghana			0.136	0.163
2000	9.90	12.17		
2010	8.40	9.15		
Indonesia			0.136	0.163
2000	6.93	7.17		
2010	3.10	3.22		
USA			0.136	0.163
2000	1.84	1.43		
2010	1.55	1.21		

Nigeria: As Table 2 shows, the values of the computed Kolmogorov-Smirnov test statistic for the both censuses and for both sexes in Nigeria exceeded the critical value (0.163) at one percent (1%) level of significance. Thus, the null hypothesis is rejected indicating that the distribution of population by age in single years among 10 end digits is not uniform for Nigerian censuses. That is, it is not free from error of digit preference/avoidance. This has confirmed the results of (Ohaegbulam, 2005 and Nwogu, 2006; 2011) which showed that the qualities of age and sex data in Nigeria are quite low and have a common error of age heaping at ages 0 and 5 while other digits are avoided.

Ghana: Table 2 also shows that the values of the computed Kolmogorov-Smirnov test statistic for Ghana are significant at 1% levels of significance indicating that discrete uniform distribution does not provide adequate fit to the distribution of Ghanaian population among the 10 end digits. This again, shows that the distribution of the population by age in single years is not completely free from digit preference/avoidance error.

Indonesia: Also, from Table 2, the computed values of the Kolmogorov-Smirnov test statistic are significant at 1% levels of significance indicating that the population is not uniformly distributed among the 10 end digits which in turn shows that the distribution of the population by age in single years is not completely free from digit preference/avoidance error.

United State of America: The quality of age-sex distribution of population for both censuses and sexes of the US is usually adjudged reliable following the relatively low value of the RI. However, the computed value of the Kolmogorov-Smirnov test statistic shown in Table 2, are significant at 1% levels of significance. This indicates that distribution of population among the 10 end digits is not uniform. This, in turn, shows that the distribution of population by age in single years is also not free from error of digit preference/avoidance in the US census population.

4 Summary, Recommendation and Conclusion

In summary, quality of demographic data and Goodness-of –Fit of the discrete uniform distribution to the distribution of the populations by age in single years among end digits in Nigeria, Ghana, Indonesia and United State of America have been discussed in this paper. When the type and source of demographic data is understood, it goes ahead to enable the user to determine the nature and degree of adjustment to which defective data may be subjected and serves as an aid to improved data quality in future surveys. The ultimate objective of this study is to assess the validity of assumption that when data on population by age in single years is correctly reported then the distribution of population among end digits is uniform. The data used are secondary data derived from the census populations reported by age in single years from Nigeria, Ghana, Indonesia and United State of America. Measures of digit preference/avoidance most commonly used are the Whipple's index (WI), Digit Preference Index (DPI), Myer's Index (MI) and the Ramachandran Index (RI). However, in this study, the Ramachandran Index (RI) was used because the obvious advantages it has over others. The Kolmogorov-Smirnov test was used to test the Goodness-of-Fit of the uniform distribution to the distribution of population among end digits in the selected countries.

The results of the analyses show that the null hypotheses were all rejected at 1% levels of significance, indicating that the distribution of population among the 10 end digits is not uniform for both sexes in all the selected countries. In conclusion, the distribution of population by age in single years is not completely free from digit preference/avoidance in all the selected countries. The study therefore recommends that the age data should be adjusted for the error of digit/avoidance before using them for further estimation of demographic parameters or at least used with caution.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

Competing Interests

Authors have declared that no competing interests exist.

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APPENDICES

Test of goodness-of-fit of Discrete Uniform Distribution to the Distribution of Population among End-digits

Appendix A1. Nigeria 1991

Digit	1991				1991			
	Male				Female			
	% $B_v(i)$	**$F_o(x)$**	**$S_n(x)$**	**$\|F_o(x)-S_n(x)\|$**	**% $B_v(i)$**	**$F_o(x)$**	**$S_n(x)$**	**$\|F_o(x)-S_n(x)\|$**
0	32.76	10	32.76	**22.76**	36.64	10	36.64	**26.64**
1	5.37	20	38.13	18.13	4.40	20	41.04	21.04
2	8.36	30	46.49	16.49	7.65	30	48.69	18.69
3	5.58	40	52.07	12.07	4.86	40	53.55	13.55
4	4.81	50	56.88	6.88	4.25	50	57.80	7.80
5	19.65	60	76.53	16.53	20.10	60	77.90	17.90
6	5.45	70	81.98	11.98	5.03	70	82.93	12.93
7	5.83	80	87.81	7.81	5.08	80	88.01	8.01
8	8.04	90	95.85	5.85	8.36	90	96.37	6.37
9	4.14	100	100.00	0.00	3.62	100	100.00	0.00
	100.00				100.00			

$D_{cal} = Max|F_0(x)-S_n(x)| = 22.76$ $\quad$ $D_{cal} = Max|F_0(x)-S_n(x)| = 26.64$

Appendix A 2. Nigeria 2006

Age	2006				2006			
	Male				Female			
	% $B_v(i)$	**$F_o(x)$**	**$S_n(x)$**	**$/F_o(x)-S_n(x)/$**	**% $B_v(i)$**	**$F_o(x)$**	**$S_n(x)$**	**$/F_o(x)-S_n(x)/$**
0	29.91	10	29.91	**19.91**	30.90	10	30.90	**20.90**
1	5.56	20	35.47	15.47	5.13	20	36.03	16.03
2	9.04	30	44.51	14.51	8.81	30	44.84	14.84
3	6.13	40	50.64	10.64	5.86	40	50.70	10.70
4	5.71	50	56.35	6.35	5.33	50	56.03	6.03
5	17.51	60	73.86	13.86	18.11	60	74.14	14.14
6	6.24	70	80.10	10.10	5.89	70	80.03	10.03
7	6.13	80	86.23	6.23	6.09	80	86.12	6.12
8	8.56	90	94.79	4.79	8.91	90	95.03	5.03
9	5.21	100	100.00	0.00	4.98	100	100.00	0.00
	100.00				100.00			

$D_{cal} = Max|F_0(x)-S_n(x)| = 19.91$ $\quad$ $D_{cal} = Max|F_0(x)-S_n(x)| = 20.90$

Appendix B1. Ghana 2000

Age	2000				2000			
	Male				Female			
	% $B_v(i)$	$F_o(x)$	$S_n(x)$	$/F_o(x)-S_n(x)/$	% $B_v(i)$	$F_o(x)$	$S_n(x)$	$/F_o(x)-S_n(x)/$
0	19.90	10	19.90	**9.90**	22.17	10	22.17	**12.17**
1	7.03	20	26.93	6.93	6.318	20	28.48	8.48
2	11.09	30	38.02	8.02	10.94	30	39.42	9.42
3	8.40	40	46.42	6.42	7.77	40	47.19	7.19
4	7.95	50	54.37	4.37	7.63	50	54.82	4.82
5	14.87	60	69.24	9.24	15.31	60	70.13	10.13
6	8.27	70	77.51	7.51	8.12	70	78.25	8.25
7	6.98	80	84.49	4.49	6.46	80	84.71	4.71
8	9.33	90	93.82	3.82	9.46	90	94.17	4.17
9	6.14	100	100.00	0.00	5.78	100	100.00	0.00
	100.0				100.0			

$D_{cal} = Max|F_0(x)-S_n(x)| = 9.90$ $D_{cal} = Max|F_0(x)-S_n(x)| = 12.17$

Appendix B2. Ghana 2010

Age	2010				2010			
	Male				Female			
	% $B_v(i)$	$F_o(x)$	$S_n(x)$	$/F_o(x)-S_n(x)/$	% $B_v(i)$	$F_o(x)$	$S_n(x)$	$/F_o(x)-S_n(x)/$
0	18.40	10	18.40	**8.40**	19.15	10	19.15	**9.15**
1	8.75	20	27.15	7.15	8.11	20	27.26	7.26
2	10.77	30	37.92	7.92	10.65	30	37.91	7.91
3	8.67	40	46.59	6.59	8.58	40	46.49	6.49
4	8.49	50	55.08	5.08	8.33	50	54.82	4.82
5	12.99	60	68.07	8.07	13.44	60	68.26	8.26
6	8.34	70	76.41	6.41	8.26	70	76.52	6.52
7	7.99	80	84.40	4.40	7.93	80	84.45	4.45
8	9.35	90	93.76	3.76	9.39	90	93.84	3.84
9	6.23	100	100.0	0.00	6.17	100	100.00	0.00
	100.0				100.0			

$D_{cal} = Max|F_0(x)-S_n(x)| = 8.40$ $D_{cal} = Max|F_0(x)-S_n(x)| = 9.15$

Appendix C1. Indonesia 2000

Age	2000				2000			
	Male				Female			
	%B_v(i)	**F_o(x)**	**S_n(x)**	**/F_o(x)-S_n(x)/**	**%B_v(i)**	**F_o(x)**	**S_n(x)**	**/F_o(x)-S_n(x)/**
0	16.93	10	16.93	**6.93**	17.17	10	17.17	**7.17**
1	9.43	20	26.36	6.36	9.20	20	26.37	6.37
2	9.58	30	35.94	5.94	9.39	30	35.76	5.76
3	8.40	40	44.34	4.34	8.42	40	44.18	4.18
4	8.66	50	53.00	3.00	8.86	50	53.04	3.04
5	13.13	60	66.13	6.13	13.34	60	66.38	6.38
6	8.12	70	74.25	4.25	8.02	70	74.40	4.40
7	8.96	80	83.21	3.21	8.76	80	83.16	3.16
8	8.61	90	91.82	1.82	8.57	90	91.73	1.73
9	8.12	100	100.00	0.00	8.21	100	100.00	0.00
	100.00				100.00			

D_{cal} = Max|F_0(x)-S_n(x)| = 6.93 D_{cal} = Max|F_0(x)-S_n(x)| = 7.17

Appendix C2. Indonesia 2010

Age	2010				2010			
	Male				Female			
	% B_v(i)	**F_o(x)**	**S_n(x)**	**/F_o(x)-S_n(x)/**	**%B_v(i)**	**F_o(x)**	**S_n(x)**	**/F_o(x)-S_n(x)/**
0	12.85	10	12.85	2.85	12.79	10	12.79	2.79
1	10.25	20	23.10	**3.10**	10.12	20	22.91	2.91
2	9.95	30	33.05	3.05	9.88	30	32.79	2.79
3	9.33	40	42.38	2.38	9.39	40	42.18	2.18
4	10.01	50	52.39	2.39	10.18	50	52.36	2.36
5	10.70	60	63.09	3.09	10.86	60	63.22	**3.22**
6	9.11	70	72.20	2.20	9.19	70	72.41	2.41
7	9.74	80	82.94	1.94	9.61	80	82.02	2.02
8	8.81	90	90.75	0.75	8.69	90	90.71	0.71
9	9.24	100	100.00	0.00	9.29	100	100.00	0.00
	100.00				100.00			

D_{cal} = Max|F_0(x)-S_n(x)| = 3.10 D_{cal} = Max|F_0(x)-S_n(x)| = 3.22

Appendix D1. USA 2000

Age	2000				2000			
	Male				Female			
	% $B_v(i)$	$F_o(x)$	$S_n(x)$	/$F_o(x)$-$S_n(x)$/	% $B_v(i)$	$F_o(x)$	$S_n(x)$	/$F_o(x)$-$S_n(x)$/
0	11.00	10	11.00	1.00	10.80	10	10.80	0.80
1	10.37	20	21.37	1.37	10.27	20	21.07	1.07
2	10.36	30	31.73	1.73	10.28	30	31.35	1.35
3	10.11	40	41.84	1.84	10.08	40	41.43	1.43
4	9.69	50	51.53	1.53	9.68	50	51.11	1.11
5	9.94	60	61.47	1.47	9.92	60	61.03	1.03
6	9.60	70	71.07	1.07	9.67	70	70.70	0.70
7	9.70	80	80.77	0.77	9.79	80	80.49	0.49
8	9.57	90	90.34	0.34	9.68	90	90.17	0.17
9	9.60	100	100.00	0.00	9.78	100	100.0	0.00
	100.0				100.0			

$D_{cal} = Max|F_0(x)\text{-}S_n(x)| = 1.84$ $\quad$ $D_{cal} = Max|F_0(x)\text{-}S_n(x)| = 1.43$

Appendix D 2. USA 2010

Age	2010				2010			
	Male				Female			
	% $B_v(i)$	$F_o(x)$	$S_n(x)$	/$F_o(x)$-$S_n(x)$/	% $B_v(i)$	$F_o(x)$	$S_n(x)$	/$F_o(x)$-$S_n(x)$/
0	10.89	10	10.89	0.89	10.72	10	10.72	0.72
1	10.30	20	21.19	1.19	10.22	20	20.94	0.94
2	10.28	30	31.47	1.47	10.21	30	31.15	1.15
3	10.08	40	41.55	**1.55**	10.06	40	41.21	**1.21**
4	9.72	50	51.27	1.27	9.69	50	50.90	0.90
5	9.92	60	61.19	1.19	9.91	60	60.81	0.81
6	9.67	70	70.86	0.86	9.74	70	70.55	0.55
7	9.77	80	80.63	0.63	9.85	80	80.40	0.40
8	9.67	90	90.30	0.30	9.77	90	90.17	0.17
9	9.70	100	100.00	0.00	9.84	100	100.0	0.00
	100.0				100.0			

$D_{cal} = Max|F_0(x)\text{-}S_n(x)| = 1.55$ $\quad$ $D_{cal} = Max|F_0(x)\text{-}S_n(x)| = 1.21$

$$D_{0.05,\ 100} = \frac{1.36}{\sqrt{100}} = 0.136 \qquad D_{0.01,\ 100} = \frac{1.63}{\sqrt{100}} = 0.163$$

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